The Combinatorial Retention Auction Mechanism (CRAM)

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Abstract: We propose a reverse uniform-price auction called the Combinatorial Retention Auction Mechanism (CRAM) integrating both monetary and non-monetary incentives (NMIs). CRAM combines the cash bonus and NMIs each employee demands into a single cost parameter, retains the lowest cost employees, and provides each with an individualized compensation package whose total cost equals the cost of the first excluded bid. We provide an optimal bidding strategy that is dominant strategy incentive compatible, characterize which employees are better off or worse off, demonstrate the potential cost savings, and document the increase in social welfare from utilizing CRAM instead of cash compensation alone.

Keywords: Market Design; Combinatorial Auction; Labor Markets; Compensation; Defense Economics.

JEL classification: D44, D47, J33, J45

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0. Introduction

We propose a mechanism called the Combinatorial Retention Auction Mechanism (CRAM), which allows an organization to utilize both monetary and non-monetary incentives (NMIs) for employee retention, selection, or promotion. CRAM is a reverse uniform-price multi-item auction in which a single employer identifies the lowest-cost subset of employees to hire, retain, or promote. By incorporating a continuous monetary incentive into the traditional combinatorial auction environment in which bidders have discrete, and possibly interdependent, valuations for multiple non-monetary goods or services, we are able to design a mechanism that not only achieves uniform cost outcomes, but which is also dominant strategy incentive compatible.

The mechanism's implementation is straightforward: CRAM elicits employees' reservation values by asking them the minimum monetary wage or bonus and set of NMIs they require to be selected for employment or promotion. For each employee, the cost of providing the requested cash incentive in addition to the set of NMIs is calculated and presented as a single cost parameter. Then, CRAM selects any specified number of lowest-cost employees, with each selected employee receiving the set of NMIs he or she requested along with a cash incentive at least as large as the monetary amount requested. In particular, each selected employee receives a monetary incentive equal to the total cost of the first-excluded bid (from the lowest-cost employee not selected) minus the total cost of the selected employee's set of NMIs received. Therefore, as would be the case with a standard uniform-price monetary auction, the cost of each selected employee equals the cost of the first excluded employee.

CRAM is dominant-strategy incentive-compatible; it is weakly dominant for each employee to reveal his or her true reservation value and true NMI values by (1) choosing the set of NMIs that maximizes the difference between the employee's value of NMIs and the employer's stated cost of NMIs and (2) announcing the minimum additional monetary incentive required for voluntary selection.

In what follows, we will primarily discuss the application of CRAM in the context of employee retention, although the mechanism can also be applied to the selection of employees and the design of compensation packages for purposes of hiring or promotion. A general but a brief discussion about combinatorial auctions and the application of CRAM to the U.S. Department of Defense (DoD)'s retention problem will be presented in the following sections. Readers with sufficient background in either field may choose to skip over the particular section.

Our contributions are as follows: First, we provide the general framework, characterization, and properties of CRAM. CRAM provides a simple and straightforward way of determining the retention cost of each employee, which set of employees should be retained, and which benefits (monetary and non-monetary) should be provided to each employee. This process lessens the burden on the employer (who acts as the auctioneer under CRAM) as well as the employees (who are the bidders or auction participants). We provide an optimal bidding strategy for the employee that is dominant strategy incentive compatible and show that any optimal bidding strategy must take this form. Each employee maximizes his welfare or utility by selecting the set of NMIs that maximizes his total surplus (total value minus total cost for the entire NMI combination), and specifying the additional cash compensation he requires (in addition to his chosen NMIs) to be retained. The cash amount bid plus the value of the NMI combination chosen should equal the reservation value for each employee.

Next, we show that the cost of retaining employees via CRAM is less than or equal to the cost of retaining employees under even the most efficient and cost-effective cash compensation system: A monetary retention auction. This result is driven by the fact that CRAM takes advantage of the surplus generated by offering NMIs that employees may value more than the cost to provide. Thus, under CRAM, the employer can potentially provide an employee the same value as a cash incentive but at a lower cost by incorporating NMIs. Note, moreover, that the CRAM approach is actually a more general framework in which a monetary retention auction is simply a special case of CRAM (in which the set of NMIs offered is empty).

Because CRAM with NMIs may retain a different set of individuals compared to the monetary auction, determining whom the mechanism benefits is not straightforward. Therefore, we also compare an employee's utility under CRAM to utility under the monetary retention auction and show which sets of employees are better off and which are worse off under various conditions. The employees are broken into four sets. An employee not retained under either mechanism is indifferent. An employee retained under the monetary retention auction and not CRAM is better off under monetary retention auction because he receives compensation greater than the reservation utility when retained. Similarly, an employee retained under CRAM but not under the monetary retention auction prefers CRAM. For anyone who is retained under both mechanisms, he may be better or worse off depending on the amount of change in the cost per employee and the surplus (value minus cost) the employee generates through the NMIs. An employee retained under both mechanisms will be better off under CRAM if the cost per employee does not drop by more than his gain in surplus from the NMIs.

Finally, we compare the social welfare, the sum of both retained and not retained employees' utility minus employer's cost, and show that social welfare is (weakly) greater under CRAM than under the monetary retention auction.

There are two important caveats to keep in mind. First, the approach of our work is in line with practical market design (Roth 2002) rather than with optimal mechanism design. Second, the design of CRAM is primarily motivated by efforts to improve the existing retention process used by the Department of Defense, which currently utilizes a strictly monetary bonus system for such purposes. In particular, we developed CRAM to reduce retention cost, accurately retain the desired number of service members, and improve the efficiency of NMI distribution.

I. Combinatorial Auctions

A. General Characteristics and Applications

Combinatorial auctions generally deal with bidding on multiple objects. What makes combinatorial auctions interesting and difficult is the computational complexity. With n goods introduced, there are 2^n possible combinations of goods (including the empty set) that the auctioneer and the participants may have to consider. Formally, these problems are considered to be NP-complete, meaning that typical computers may have difficulty finding an "optimal solution."

While combinatorial auctions have always been of interest, the field has seen the greatest growth with its application to the Federal Communication Commission (FCC) spectrum auctions. Between 1994 and 2003, the FCC utilized some form of combinatorial auction 41 times, raising over $40 billion in revenue (Kwasnica, Ledyard, Porter, and DeMartini 2005). Even prior to this highly publicized utilization by the FCC, combinatorial auctions had been employed to enhance market and non-market transactions by public and private entities. Grether, Isaac, and Plott (1981) were one of the earlier proposers of using an auction type of design to solve airport time slot allocation problems for the FAA. Rassenti, Smith, and Bulfin (1982) further improved the use of a computer-assisted smart market way of solving the landing rights problem. Banks, Olson, Porter, Rassenti, and Smith (2003) documented various combinatorial auctions that have been utilized to solve complicated government and non-government allocation problems, including use for energy trading by the Arizona Energy Exchange, gas delivery by the Federal Energy Regulatory Commission, payload manifest for Space Shuttle, resource allocation for Cassini mission to Saturn, train scheduling, transportation services, pollution markets, and markets to exchange financial portfolios.

B. Unique Challenges of Combinatorial Auctions

Such combinatorial auctions face problems not usually encountered with single-object auctions, including the following challenges detailed by Pekeč and Rothkopf (2003):[4]

The Exposure Problem – First, multi-item auctions are subject to the exposure problem, in which bidders face the risk of winning unwanted items. For example, consider an auction of two items *a* and *b*. If these two items are perfect complements (such that *a* and *b* only have positive value when combined) then, if bids on the combination $\{a, b\}$ are not allowed, bidders are exposed to the risk of winning only one (worthless) item if they must submit separate bids on the individual items. Alternatively, if the two items are perfect substitutes (such that a bidder would like to win *a* or *b* but not both) then bidders might be exposed to the risk of winning both items when only one or the other is wanted.

The Communication Complexity Problem – The exposure problem above suggests that, whenever the items being auctioned are complements or substitutes (to any degree), it may be insufficient for the auction to accept only bids on individual items. Instead, bidders must potentially submit bids on all possible combinations of items he or she might receive. Even with a relatively small

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[4] Pekeč and Rothkopf (2003) also discuss the "threshold problem," but this concern is not detailed here as it is not particularly relevant in the present context. Other authors also add "jump bidding" as a problem for combinatorial auctions but, as discussed by Isaac, Salmon, and Zillante (2004), jump bidding is not necessarily a disadvantage from the auctioneer's perspective and, in fact, can increase revenue.

auction involving only 7 items, for example, this could potential demand as many as $2^7-1 = 127$ bids from each bidder. Thus, combinatorial bidding is highly complex in terms of the communication, calculation, and even cognition required.

The Winner Determination Problem – It can be much more difficult to determine winners in a combinatorial auction because, unlike a single-item auction, the highest bid on a single (or package of items) is not guaranteed to win that item (or package). This is because some alternative combination of bids might generate higher revenue. Finding this revenue-maximizing alternative combination of bids is also not a simple problem. Further problems arise with tie breaking rules. How are ties to be broken? Which group of people are winners if different combinations of goods by different sets of participants yield the same revenue? What if the highest revenue generating combination does not utilize all possible resources?

Depending upon the auction design, any or all of the above problems might also be present to some degree in the current context of bidding on combinations of non-monetary incentives (NMIs). As we will discuss, however, the particular format of bid submission we propose can circumvent both the exposure and communication complexity problems while favorable features of the environment (and, in some cases, reasonable simplifying assumptions) greatly minimize the winner determination problem.

C. Established Combinatorial Auction Formats

Within the combinatorial auction family, the following are some auction formats that have drawn considerable attention:

1. Simultaneous Multiple Round Auction (SMR): An auction format utilized by the FCC which does not allow for package bidding, SMR is often used as a benchmark comparison to other combinatorial auction designs.

2. Adaptive User Selection Mechanism (AUSM): Developed by Banks, Ledyard, and Porter (1989), AUSM allows for package bidding in continuous time.
3. Resource Allocation Design (RAD): Developed by Kwasnica, at el. (2005), RAD is a hybrid of SMR and AUSM incorporating an additional pricing feature to guide bidders.
4. Combinatorial Clock Auction (CCA): Developed by Porter, Rassenti, Roopnarine, and Smith (2003), CCA uses a "clock" as a guide for bidding (similar to an English auction).
5. Simultaneous Multiple Round Package Bidding (SMRPB): Developed by the FCC as a variant of RAD, SMRPB includes the ability to utilize an "exclusive OR" bidding function.

We do not include the details of these auction mechanisms here, but interested readers may consult the provided references. Brunner, Goeree, Holt, and Ledyard (2010) summarized some of the commonly discussed combinatorial auctions mentioned above and compared their performance via experiments. Brunner et al. (2010) found that, when complementarities are present, package bidding is recommended and that CCA generally yields the highest revenue.

With the wide variety of combinatorial auction mechanisms proposed and employed, it is natural to ask whether the classic Vickrey-Groves-Clarke (VCG) mechanism for single-item auction design (Vickrey 1961, Clarke 1971, Groves 1973) could be generalized and applied to bidding for combinations of items. Such a "combinatorial VCG mechanism" can indeed be generated and even retains the desirable features of allocative efficiency and truth revelation among bidders. As Pekeč and Rothkopf (2003) explain, however, VCG mechanisms are both impractical and unattractive – and therefore rarely used – in the combinatorial context, because such mechanisms involve refunding to bidders the increase in value caused by their bids. Since these "refunds" can be significant, the VCG mechanism is "revenue deficient." In addition, such mechanisms may be subject to manipulation by either bidders or bid-takers (via collusion, insincere bidding, or "false name" bidding).

D. Application to Retention and Non-Monetary Incentives

Due to institutional restrictions, the specific auction mechanisms mentioned above cannot be directly or easily adapted and applied to the retention problem. First, the above auction formats are "forward" auctions that primarily deal with selling objects. Procurement auctions, or "reverse" auctions, are auctions where the auctioneer is interested in buying goods and services instead of selling. Therefore, retention auctions are more similar to such procurement auctions. There are many differences between procurement auctions and retention auctions, however, again due to institutional features.

For example, in procurement auctions, buyers can procure half of the goods and services, or split the award among multiple providers in order to keep the bidders competitive[5] (Chaturvedi, Beil, and Martinez-de-Albeniz 2013). In the active-duty military, however, it is not feasible to retain a portion of a person. Furthermore, NMIs are specific incentives that are salient for compensating employees but may not be salient in procurement or the standard forward auctions. The next section will further discuss some characteristics and institutional features that require changes to the known combinatorial auctions and the reason for developing CRAM.

Finally, it is worth noting that a combinatorial auction can be an extremely useful tool for aggregating information, as well as endogenously determining a market-clearing price. When the designer lacks information on which NMIs may

[5] Awarding the procurement to only one vendor may make that vendor a monopoly in the future due to technological advancement.

or may not be sub or super modular, it may be best left for each bidder to choose the optimal set of his own NMIs.

For example, a bidding employee may value the NMI of geographic stability highly, but only in combination with another NMI which provides him the job location of his choice (such that the two NMIs are complements). Another bidder might assign high value either to a more flexible work schedule or to the opportunity to telework, but may not ascribe a relatively high value to the combination (such that the two NMIs are substitutes).

Allowing each bidder (rather than the bid-taker) to select for himself the combination of NMIs that generates the most value (relative to cost) greatly simplifies the information aggregation task of the mechanism. Moreover, when it comes to price formation, instead of exogenously estimating the price with a large room for error, these auctions will endogenously determine the market-clearing price.

II. Application to the U.S. Department of Defense

There is no more natural and valuable application for CRAM than to the retention problem within the armed forces. With over 1.4 million active duty and 1.1 million Reserve and National Guard service members in 2013 (DoD 2013), the U.S. military's labor force is not only distinctive in its size but also because it is internally grown, with strictly internal promotions and no external hires beyond entry-level positions. For example, if an Admiral retires and the Navy is in need of a replacement, it cannot simply go to the general labor market and hire a new Admiral from another organization. Instead, the Navy must promote from within. Therefore, the DoD and each of the services must carefully plan its force structure over the long term, with precise retention and promotion decisions being extremely critical force-management activities.

In terms of budget and compensation, approximately 51.4 percent of military compensation is cash compensation, while 20.5 percent of military compensation involves non-cash items (such as education and health care benefits), and 28.1 percent of the compensation is deferred compensation (such as retirement pay accrual) (DoD 2012). Out of the $525 billion budget for the DoD in 2012, $181 billion was related to pay and benefits for military personnel (Harrison and Montgomery 2011). With cuts in the defense budget, however, the DoD also needs to find savings in its cost of compensation.

Special and Incentive (S&I) pays are authorized by law to provide the military services the flexibility needed for recruitment, retention, and separation. (OSD Military Compensation 2013). There are currently over 60 authorized S&I pays. These pays can be significant. Examples include: 1) Selective Reenlistment Bonus (SRB), which authorizes the services to pay up to $90,000 for a minimum three-year reenlistment; 2) Surface Warfare Officer Continuation Pay that authorizes the Navy to pay up to $50,000 to eligible officers for committing to a Department Head tour;[6] and 3) Critical Skills Retention Bonus (CSRB), which authorizes up to $200,000 over a service member's career[7] for a skill-specific retention. Some S&I pays are much smaller, such as Demolition Duty Pay – a hazardous duty, which adds $150 per month for the assignment's duration. Of course, these S&I pays are reserved for very select groups of service members during a shortage of manpower.

To provide perspective on a service member's base cash compensation during the 2013 calendar year, excluding S&I pays, an average Staff Sergeant in the U.S. Army (pay grade E-6) with 10 years of total service and three dependents

[6] Department Head tour is a type of leadership tour for the Navy's ship drivers.
[7] US military service members are typically eligible for full retirement at 20 years of service.

would get annual cash compensation[8] of $60,520.08. An average Captain in the U.S. Marine Corps (pay grade O-3) with five years of total service and no dependents would get annual cash compensation of $80,107.68 (DFAS 2013). Therefore, these S&I pays can be a significant portion of the service member's overall income.

The CRAM is designed to support DoD's retention process. CRAM is developed to improve control in 1) reducing retention cost, 2) accurately retaining the proper number of service members, and 3) improving the efficiency and effectiveness of distributing NMIs. The DoD has been limited to utilizing a posted-price format for providing the S&I bonuses mentioned above, including selective reenlistment bonuses.[9] Furthermore, these bonuses are provided as purely monetary compensation, thus forgoing any surplus that may be gained by incorporating NMIs as well.

Coughlan, Gates, and Myung (2014), CGM henceforth, described the additional surplus that the DoD can potentially gain by providing personalized NMI packages. Furthermore, CGM stressed the importance of utilizing NMIs, the difficulty and inefficiency of providing a universal incentive package[10] of NMIs, as well as the extreme variability in preference for NMIs by service members across and within communities. CGM found that, although none of the NMIs examined provided significant value to at least 50 percent of the service members surveyed, approximately 80 percent of service members expressed a significant value for at least some NMIs.

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[8] Cash compensation is the Basic Pay (salary) plus additional cash payments for housing and allowance for subsistence. In addition, there are deferred and universal compensation elements, such as health insurance and tax advantages, but we do not include these in computing cash compensation.

[9] Posted-price format implies that the Service announces the bonus amount and the market determines how many service members accept the announced bonus. This method lacks control over the quantity of service members accepting the bonus and can be expensive if too many service members accept.

[10] Universal Incentive Package means that everyone receives the same set of NMIs.

As with designing any market, the market designer must consider important normative and positive characteristics that the market user may value. For example, Pekeč and Rothkopf (2003) discussed that some of the key considerations when designing a combinatorial auction are allocative efficiency, cost minimization, low transaction cost, fairness, failure freeness, and transparency.

In addition to the aforementioned considerations, our market design for the DoD emphasizes the following normative characteristics as critical features of any rectention mechanism:

1. Egalitarian – Perception of equality: The military is of a strong mindset that service members of equal rank and position should get equal pay. Hence, in terms of S&I pay, everyone receiving the specific S&I bonus should get the same bonus amount.
2. Dominant strategy incentive compatible mechanism – Transparency and ease of use: The military prefers a mechanism that is easy to understand and minimizes strategic gaming by the participants.
3. Low transaction cost – Minimal time required of auction participants: This consideration is different from many other combinatorial auction designs. Unlike the FCC auction, which can take a form of ascending bid auction requiring participants to observe and interact for hours or days at a time, this is not feasible for the DoD. Different service members may be involved in operational activities throughout the world. A submariner may be undersea for an extended period of time and only have one chance to submit a single bid. An airman may be deployed in a hostile environment and unable to frequently check the current auction market status. Therefore, conducting a simultaneous ascending bid or clock type auction is not practical.

Taking these features into consideration, we believe that CRAM is optimally designed for the dual problems of retention and distribution of NMIs within the DoD.

III. Outline

We describe the general environment for CRAM in Section IV and formally define CRAM in Section V. Section VI discusses the employee's optimal bidding strategy. Section VII introduces the monetary retention auction as a benchmark against which CRAM's characteristics are compared. Section VIII compares the employer's cost under CRAM to the monetary retention auction, Section IX explores employee utility, and Section X compares total social welfare under the alternative mechanisms. We end with conclusions in Section XI.

IV. The Environment

A. The Retention Problem

Let I be a set of employees currently qualified for and seeking retention with a given employer. The employer will retain $q \leq |I|$ of these employees. Each employee $i \in I$ ultimately retained by the employer will receive a monetary incentive, $m_i \in \mathbb{R}$, as well as some combination of NMIs in his or her ultimate compensation package.

Denote by N the set of all NMIs offered by the employer and by $S_i \subseteq N$ the subset of NMIs potentially received by employee $i \in I$. Each employee can consume at most one of each of the $|N|$ NMIs. Therefore, there are $2^{|N|}$ different potential combinations of NMIs an employee could receive (including the empty set). We assume that each NMI is a non-rivalrous but excludable good (thus, each is a club good).

B. Employee Preferences

Each employee i is endowed with a NMI valuation function $v_i : S \subseteq N \to \mathbb{R}$ that calculates a dollar-equivalent value for any combination of NMIs. Each employee i's utility for any combination of a monetary incentive (m_i) and a non-monetary incentive package (S_i) can then be expressed by a quasi-linear utility function $U_i(m_i, S_i) = v_i(S_i) + m_i$.

We normalize $v_i(\emptyset) = 0$ for all $i \in I$. Note that we also explicitly allow for an employee's valuation, $v_i(S_i)$, of any combination of non-monetary incentives to be modular, submodular, or supermodular with respect to the valuations of the individual NMIs included in that combination. In other words, the NMIs within a package might be complements or substitutes or some combination thereof.

Each employee $i \in I$ is further endowed with a reservation value $r_i \in \mathbb{R}$, which reflects the employee's opportunity cost of being retained by the employer (or, alternatively, the employee's "willingness-to-retain" or the expected value of the employee's outside offer or opportunity). If not retained by the employer, each employee i will enjoy utility r_i.[11] Each employee i's reservation value, r_i, and NMI valuation function, v_i, are private information.

It should be noted that the productivity of employee i is assumed to be independent of both his reservation value, r_i, and his NMI valuation function, v_i.

[11] Note that we do allow for the possibility that $r_i < 0$, such that the employee would actually be willing to pay the employer or take a pay cut to be retained.

Hence, the employer is indifferent over which employees are actually retained, as long as $q \le |I|$ employees are retained and that the cost of doing so is minimized.[12]

We denote the final compensation package, consisting of cash and a set of NMIs, given to any retained employee *i* as $P_i = (m_i^*, S_i)$. Employee *i*'s utility for this final retention package is then given by $U_i(P_i) = U_i(m_i^*, S_i) = v_i(S_i) + m_i^*$.

C. Employer Costs

For each individual NMI, $s^n \in S \subseteq N$ with $n \in \{1,2,...,|N|\}$, the employer's cost to provide that particular NMI to any individual employee, $cost(s^n)$, is public knowledge (or at least communicated to all employees prior to the retention decision).[13] Note that, in the operation of the mechanism, an NMI's cost ultimately and simply reflects the "price tag" charged to any employee who receives that NMI. Therefore, the employer is actually free to publish (and utilize in CRAM) a cost that is higher than its actual estimate of unit cost, perhaps to hedge against perceived risk due to cost or demand uncertainty. The model and mechanism can also easily generalize to the scenario in which the cost of each NMI might vary across employees – such that the employer's cost to provide NMI s^n to employee *i* would be given by $cost_i(s^n)$ – but, for simplicity, we assume uniform NMI costs in the discussion that follows.

Because each NMI is a non-rivalrous club good, provision of each NMI is characterized by constant marginal cost.[14] Hence, the cost to provide any given

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[12] A model incorporating observable quality in a retention auction framework is introduced by Myung (2013).

[13] The alternative "bid-method" CRAM design, described later in Appendix C, does not require disclosure of NMI costs.

[14] Note that the assumption of constant marginal cost also implies the absence of quantity constraints, such that each NMI could conceivably be provided to all retained employees. If,

NMI, s^n, to any given number of employees, *x*, is simply given by $x \bullet cost\left(s^n\right)$. Thus, there are neither economies nor diseconomies of scale in providing any particular NMI. We normalize $cost(\varnothing)=0$. We further assume that there are neither economies nor diseconomies of *scope* in providing any *combination* of NMIs. That is, the total cost to provide any set of NMIs, *S*, is given by $cost(S)=\sum_{s^n \in S} cost(s^n)$.[15]

Therefore, the employer's total cost to provide a final retention package $P_i=\left(m_i^*, S_i\right)$ to any retained employee *i* is given by $cost(P_i)=m_i^*+cost(S_i)$ or $cost(P_i)=m_i^*+\sum_{s^n \in S_i} cost(s^n)$.

D. NMI Surplus

With this understanding of employee preference and employer cost, it is helpful to define the employee NMI surplus (value in excess of cost). For any bidder *i* and any set of NMIs *S*, let $surplus(i,S)=v_i(S)-cost(S)$.

Note that, for a given set of NMIs, *S*, $surplus(i,S) \in \mathbb{R}$ is not necessarily positive. The following lemma guarantees, however, that employee NMI surplus will neither be negative for all sets of NMIs nor positive for all sets of NMIs.

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instead, binding quantity constraints are present, the published cost of any over-demanded NMI could simply be raised and those employees who have selected that particular NMI could be asked to revise their NMI selections based on the new, higher cost. This process could be repeated as necessary until the quantity of each NMI demanded no longer exceeds the quantity available. Alternatively, the "bid-method" CRAM design described in Appendix C allows for the appropriate distribution of quantity-constrained NMIs to be determined without repeated bidding by employees.

[15] We can relax the assumption of constant marginal cost and absence of economies of scale or scope in providing NMIs, however this adds computational complexity without adding significant value to the introduction of our mechanism.

LEMMA 1: For any set of available NMIs N and any employee i, $\max_{S\subseteq N}\left(surplus\left(i,S\right)\right)\geq 0$ and $\min_{S\subseteq N}\left(surplus\left(i,S\right)\right)\leq 0$.

With this notion of NMI surplus, it is instructive to note that an employee i's utility for the final retention package can now be written as $U_i(P_i)=U_i(m_i^*,S_i)$ $= v_i(S_i)+m_i^* = m_i^*+cost(S_i)+surplus(i,S_i) = cost(P_i)+surplus(i,S_i)$. Hence, employee i's utility for the final retention package is simply the employer's cost to provide that package plus the employee's NMI surplus.

Before proceeding, however, it is important to distinguish the notion of NMI surplus from total employee surplus (or supplier surplus). Recall that each employee $i\in I$ has a reservation value $r_i\in\mathbb{R}$ that reflects the employee's opportunity cost of being retained by the employer. While an employee's NMI surplus reflects how much that employee values a set of NMIs above and beyond the cost of providing those NMIs, employee total surplus reflects the utility for a total compensation package above and beyond that employee's reservation value. Hence, total employee surplus for any retained employee is equal to $U_i\left(P_i\right)-r_i=$ $v_i\left(S_i\right)+m_i^*-r_i$.

V. Combinatorial Retention Auction Mechanism (CRAM)

A. Mechanism

We formally outline the mechanism in this section, with detailed explanation of the mechanism to be provided in the subsections that follow. We also include two examples of the mechanism in Appendix C. First, define $B_i=(m_i,S_i)$ as the message or strategy being submitted by the employee to the employer, where m_i is the monetary incentive and S_i is the employee's requested set of NMIs. The employer's cost of providing B_i is $b_i=cost(B_i)=m_i+cost(S_i)$.

Without loss of generality, let $b_i \leq b_j$ if $i < j$ for all $i, j \in I$ and let b^* represent the $(q+1)$ lowest cost bid, or $b^* = b_{q+1}$. The CRAM mechanism $\Gamma = (B_1, ..., B_I, g(\bullet))$ is a collection of $|I|$ bids, $\{B_1, ..., B_{|I|}\}$ and an outcome function $g : B_1 \times ... \times B_I \to X$, where the outcome determines the retention and the compensation package in the following manner:

$$P_i = \begin{cases} (m^*_i, S_i) \text{ if } i \leq q \text{ and } b_i \leq b^* \\ (0, \varnothing) \text{ if } i > q \text{ and } b_i \geq b^* \end{cases} \quad \text{and retained if } i \leq q$$

where $m_i^* = b^* - cost(S_i)$. Therefore, persons with bid such that $i > q$ are not retained and receive their reservation value.

B. Employee Bids

Informally, the CRAM bidding process can be separated into two decision elements for each employee: (1) selecting NMIs and (2) submitting a minimum monetary incentive (or cash compensation) the employer must provide to retain that employee.

For the first decision element, employees must choose which NMIs they desire from a "menu" in which each NMI has an associated (known) cost. As we will detail below, the employer will add the cost of each NMI selected to the employee's monetary incentive request to determine the cost of retaining that employee. Thus, the NMI cost, and not just its value to the employee, factors into the employee's decision regarding which combination of NMIs to select from the menu.

The second decision element of the bidding process involves requesting a monetary incentive or cash compensation incentive. Because retained employees receive each and every NMI they have chosen from the menu, the monetary incentive bid reflects the minimum cash amount an employee must receive in order to remain voluntarily employed, conditional on the fact that the retained employee will also receive all NMIs selected.

Thus, the CRAM bidding process can be formally described as follows: Each employee i submits a bid of the form $B_i = (m_i, S_i)$, where m_i is the monetary incentive and S_i is the combination of NMIs that employee i requests to be retained. Let $B = (B_1, B_2, \ldots, B_{|I|})$ be the set of all submitted employee bids. Further, let $B_{-i} = (B_1, B_2, \ldots, B_{i-1}, B_{i+1}, \ldots, B_{|I|})$ denote the set of bids submitted by all employees other than employee i, or employee i's competing bid set.

C. Employee Cost and Retention

To retain employee i who has submitted bid $B_i = (m_i, S_i)$, the employer must provide that employee the set of NMIs, S_i, and cash compensation of at least m_i. Thus, the minimum cost to retain that employee is $b_i = cost(B_i) = m_i + cost(S_i)$.

The employer will retain the least expensive set of q employees. In other words, the employer will retain those q employees who submit the q lowest-cost bids. That said, note that, for all of the results in this paper, it is not necessary that q, the number of employees to be retained, be (1) known by the employees, (2) communicated to the employees, (3) estimable by the employees, or (4) even determined in advance by the employer prior to bid submission. In fact, given the nature of the mechanism, the employer could even choose the number of desired retainees after observing the bids submitted and calculating the total cost of various levels of retention. Allowing the employer to do so would have no effect on the optimal bidding strategy, the cost savings, or the social welfare implications of the mechanism.

Without loss of generality, let the employees be labeled and ordered such that $b_i \le b_j$ if $i < j$ for all $i, j \in I$. The employer will then retain employee *i* if and only if $i \le q$. Any employee *i* with $i > q$ will not be retained and thus receives his reservation value, r_i.

Note that a "tie" at the margin is possible, such that there exist more than one set of *q* lowest-cost bids. Whenever this occurs, it means that $b_q = b_{q+1}$ and, quite possibly, that other employees have also submitted bids that all have the q^{th} lowest cost.[16] For our analysis, we will randomly break ties, with some employees who submit a bid with the q^{th} lowest cost being retained, while others who submit bids of the same cost not being retained. Note that many of our results will hold with weak inequality because we are allowing for ties at b_q and b_{q+1}. If we do not allow for ties, our results would be stronger and hold with strict inequality.

D. Compensation for Retained Employees

Because CRAM is a uniform-price auction mechanism, all retained employees will receive a total retention package of uniform cost to the employer. In particular, each retained employee will receive a retention package whose total cost is equal to the cost of the first-excluded bid, which is the lowest-cost bid submitted among those employees not retained.

Even though each retained employee will receive a package of the same cost, however, the cash compensation and NMIs included in such packages could differ significantly across retainees. Moreover, the utility that retained employees enjoy from their compensation packages could also differ.

[16] Note that ties in bid costs above b_{q+1} or below b_q will not change the uniqueness of the set of *q* lowest-cost bids.

Given our construction that $b_i \le b_j$ if $i < j$, note that the first-excluded bid is the bid submitted by agent (q+1). We shall refer to the cost of this first-excluded bid as the "cutoff cost" and will denote this cost by $b^* = b_{q+1}$.

The employer will provide compensation package $P_i = \left(m_i^*, S_i\right)$ to any retained employee i, and the cost of this compensation package is given by $cost\left(P_i\right) = m_i^* + cost\left(S_i\right)$. Because we have specified that the compensation package for any retained employee i must satisfy $cost\left(P_i\right) = b^*$, we have that $m_i^* + cost\left(S_i\right) = b^*$ or that the cash compensation provided any retained employee i is given by $m_i^* = b^* - cost\left(S_i\right)$.

Hence, for each $i \le q$, employee *i's* retention package is given by $P_i = \left(m_i^*, S_i\right) = \left(b^* - cost\left(S_i\right), S_i\right)$. As the following lemma formalizes, it is important to recognize that every retained employee receives a monetary incentive greater than or equal to the amount requested in his or her bid.

LEMMA 2: For any employee *i* retained under CRAM, $m_i^* \ge m_i$.

Moreover, because each employee receives the exact set of NMIs requested, it is also the case that every retained employee's utility for the final retention package received will be greater than or equal to their utility from the package requested or bid.

LEMMA 3: For any employee *i* retained under CRAM, $U_i\left(P_i\right) \ge U_i\left(B_i\right)$.

E. Differences across Retained Employees

Although the cost to the employer is exactly the same for every retained employee, not every retained employee receives the same compensation package.

Different employees may have submitted different bids, $B_i = (m_i, S_i)$, requesting different NMI combinations.

Hence, if employees i and j are both retained with $B_i = (m_i, S_i)$ and $B_j = (m_j, S_j)$, these employees will receive different NMI packages whenever $S_i \neq S_j$. Furthermore, if $cost(S_i) \neq cost(S_j)$, these two retained employees will also receive different cash compensation, with $m_i^* = b^* - cost(S_i)$ and $m_j^* = b^* - cost(S_j)$.

In addition, even if two retained employees i and j do receive the exact same retention package, the utility enjoyed by these two employees will not necessarily be the same. Suppose, for example, that we have $P_i = P_j = (m^*, S)$ for these two employees. If $v_i(S) \neq v_j(S)$, then $U_i(P_i) = v_i(S) + m^* \neq U_j(P_j) = v_j(S) + m^*$, and employees i and j will experience different levels of utility despite receiving identical compensation packages.

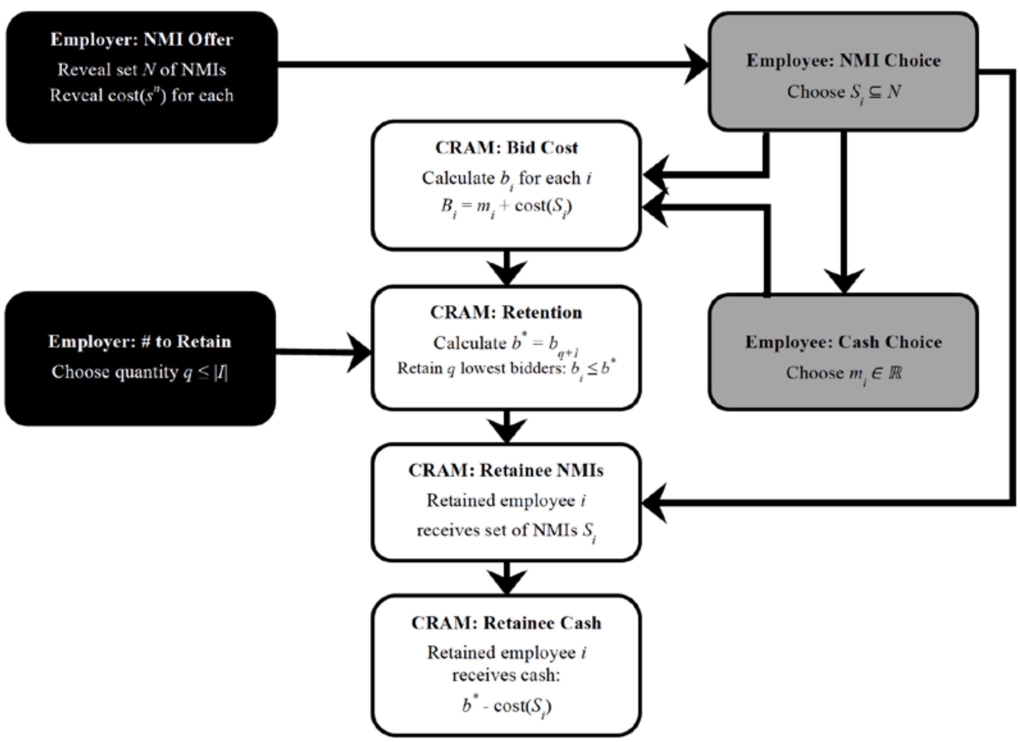
Finally, even if two retained employees do receive the same utility from their respective compensation packages, they do not necessarily enjoy the same total employee surplus, because they probably have different reservation values. Formally speaking, even if $U_i(P_i) = U_j(P_j)$, so long as $r_i \neq r_j$ we will have $U_i(P_i) - r_i \neq U_j(P_j) - r_j$ and, therefore, the two employees will receive different employee surpluses.

In sum, even though the cost of all compensation packages provided to retained employees will be the same under CRAM, (1) the NMIs received by retained employees may differ, (2) the cash compensation received by retained employees may differ, (3) the utility enjoyed by retained employees may differ, and (4) the surplus received by retained employees may differ.

F. Summary Flowchart of the Mechanism

Figure 1 summarizes the overall inputs and calculations of CRAM. Inputs from the employer are shown in the black boxes, inputs from the employees are indicated in the gray boxes, calculations performed by the mechanism are illustrated in the white boxes, while transfers of information from one stage to another stage are indicated by the arrows. In addition, two examples are included in Appendix B.

Figure 1: Summary of CRAM Inputs and Calculations



VI. Optimal Bidding Strategy

Having fully described CRAM and even begun characterizing outcomes under this mechanism, we now turn to deriving the optimal bidding strategy for employees under CRAM. We conduct this derivation in two stages, first identifying the optimal monetary bidding strategy and then identifying the optimal non-monetary bidding strategy.

A. Optimal Monetary Bidding Strategy

To understand an employee's optimal strategy for the monetary portion (m_i) of a CRAM bid, it is helpful to recall that the reservation value r_i reflects employee i's opportunity cost of being retained by the employer without any of the NMIs the employer has offered. Having selected a set of NMIs (S_i) as part of his CRAM bid, however, employee i will receive precisely those NMIs if retained by the employer.

Therefore, when determining the optimal monetary portion (m_i) of a CRAM bid, employee i must consider the revised opportunity cost of being retained with the chosen set of NMIs (S_i). Since these NMIs provide employee i a benefit of $v_i(S_i)$ if retained, the revised opportunity cost of being retained is given by $r_i' = r_i - v_i(S_i)$. In the lemma that follows, we show that employee i's optimal bidding strategy involves submitting a monetary bid that truthfully reveals this revised opportunity cost.

LEMMA 4: Given any reservation value $r_i \in \mathbb{R}$ and any set of NMIs $S_i \subseteq N$, bid $B_i = (m_i, S_i)$ maximizes employee i's utility under CRAM for any set of competing bids, B_{-i}, if and only if $m_i = r_i' = r_i - v_i(S_i)$.

B. Optimal Non-Monetary Bidding Strategy

In the previous sub-section, we demonstrated that the unique optimal monetary bid under CRAM is $m_i = r_i' = r_i - v_i(S_i)$, for any given set of NMIs

$S_i \subseteq N$. In this sub-section, we characterize the optimal non-monetary bidding strategy to accompany the now-established optimal monetary bidding strategy. In particular, we show that the optimal non-monetary bidding strategy is to select a set of NMIs S_i that maximizes employee *i*'s NMI surplus, which, recall, is given by $surplus(i, S_i) = v_i(S_i) - cost(S_i)$.

LEMMA 5: For any reservation value $r_i \in \mathbb{R}$ and any monetary bid $m_i \in \mathbb{R}$, employee *i*'s utility from being retained under CRAM, $U_i(P_i)$, will be maximized if and only if he submits a bid $B_i = (m_i, S_i)$ where $S_i \in \operatorname{argmax}_{S \subseteq N}(surplus(i, S))$.

Lemma 5 essentially says that an employee maximizes the utility of his retention package if and only if he selects a set of NMIs S_i that maximizes his NMI surplus. With our first theorem, we show that selecting such a set of NMIs, while submitting the optimal monetary bid described in Lemma 4, is the only utility-maximizing bidding strategy.

THEOREM 1: (Dominant Strategy Incentive Compatibility Theorem) Given any reservation value $r_i \in \mathbb{R}$, bid $B_i = (m_i, S_i)$ maximizes employee *i*'s utility under CRAM for any possible sets of competing bids, B_{-i}, if and only if $m_i = r_i' = r_i - v_i(S_i)$ and $S_i \in \operatorname{argmax}_{S \subseteq N}(surplus(i, S))$.

With this theorem, we have now proven that submitting a bid $B_i = (m_i, S_i)$ with $S_i \in \text{argmax}_{S \subseteq N}\left(surplus(i,S)\right)$ and $m_i = r_i - v_i(S_i)$ is the unique weakly dominant bidding strategy under CRAM.[17]

C. Costs and Utility under the Optimal Bidding Strategy

Immediately following from Theorem 1, we have two corollaries that characterize the equilibrium employee cost-to-retain and retention utility under CRAM.

COROLLARY 1: In the dominant strategy equilibrium in **Theorem 1**, the cost-to-retain associated with any employee *i* under CRAM is given by $b_i = r_i - \max_{S \subseteq N}\left(surplus(i,S)\right)$.

Corollary 1 indicates that, the greater the maximum potential NMI surplus for any employee, the lower the employee's cost-to-retain and, hence, the more likely this cost will be below the cutoff cost and therefore the more likely that the employee will be retained.

COROLLARY 2: In the dominant strategy equilibrium in **Theorem 1**, any employee *i* will receive a retention package P_i generating utility $U_i(P_i)$= $b^* + \max_{S \subseteq N}\left(surplus(i,S)\right)$ if retained.

[17] Because we have allowed for the possibility that $r_i < v_i(S_i)$, there are scenarios in which employee *i*'s optimal monetary bid, $m_i = r_i - v_i(S_i)$, will be negative. This has no impact on the results that follow. If, however, negative monetary bids were disallowed as a matter of policy, this would impact the optimal bidding strategy for any employee *i* for whom the optimal monetary bid would otherwise be negative. For example, if $r_i < 0$ but negative monetary bids are not permitted, employee *i* would be better off choosing no NMIs and submitting a monetary bid of zero, as this would be the lowest cost bid possible. Nonetheless, even in this scenario, CRAM would still retain the lowest cost employees and, so long as the "cutoff cost" b^* was sufficiently positive that there would be no changes "at the margin" (i.e., the set of individuals submitting the *q* lowest cost bids was unchanged), CRAM would also still retain the set of employees most willing to retain and the social welfare results presented later in the paper would also still hold. In other words, if negative monetary bids were disallowed, difficulties would arise only if a significant number of employees were willing to retain without any bonus whatsoever. Under such circumstances, however, there would be no need for retention incentives in the first place, rendering the entire problem addressed by this paper moot.

Corollary 2 indicates that, the greater the maximum potential NMI surplus for any employee, the greater the employee's utility if retained. In combination, these two corollaries tell us that, the greater an employee's maximum NMI surplus, the more likely it is that employee will be retained under CRAM (due to lower cost-to-retain) and the better off the employee will be (due to higher utility) if, in fact, retained.

VII. Benchmark Mechanism: Monetary Retention Auction

To evaluate the relative performance of CRAM, we compare it to the traditional method of motivating retention by offering a uniform monetary incentive to all potential retainees. In practice, the amount of any such monetary retention incentive is determined using some imperfect estimation method. However, an auction is the more cost-effective and welfare enhancing approach to setting a monetary retention incentive (and determining which employees to retain).

Therefore, let us formally describe the benchmark alternative to CRAM as a *monetary retention auction*. Furthermore, for consistency, we will consider the uniform-price auction format. Such a monetary retention auction is actually a special variation of CRAM, in which the set of NMIs $N = \{\varnothing\}$, each employee i simply submits a single monetary bid $\hat{m}_i$, and the q employees retained by the employer are those who submit the q lowest monetary bids.

With the uniform-pricing rule, each retained employee receives the same monetary retention incentive, which is set equal to the $(q+1)^{st}$-lowest bid. Let us denote the amount of this uniform monetary retention incentive – or, alternatively, the amount of the $(q+1)^{st}$-lowest bid – by $\hat{m}^*$.

It is well established that the dominant strategy in such a uniform-price monetary retention auction is for any employee i to truthfully-reveal his or her reservation value r_i by bidding $\hat{m}_i = r_i$. The monetary retention incentive, $\hat{m}^*$, received by any retained employee i will then be at least as large as the amount bid by that employee. In other words, $\hat{m}^* \geq \hat{m}_i = r_i$. Hence, each retained employee i enjoys a surplus of $\hat{m}^* - r_i$, while the total retention cost for the employer is equal to $q\hat{m}^*$.

In the sections that follow, we will compare CRAM's performance to the just-described alternative of a monetary retention auction, focusing on employer cost, employee surplus, and overall social welfare. Please recognize, however, that the cost and welfare advantages of CRAM illustrated in what follows actually underestimate its true advantages. This is because we are comparing CRAM not to retention methods actually used (a posted-price format with the price that is imperfectly estimated), but to a more efficient and less costly cash-only sealed-bid auction method.

Incidentally, because the CRAM mechanism actually addresses two human resources challenges at once – namely, (1) employee retention and (2) NMI allocation – one might ask if there is another appropriate benchmark mechanism against which we can compare CRAM in terms of the second challenge of NMI allocation. The actual method for distributing NMIs in common practice, however, is a "one-size fits all" approach: Each non-monetary incentive is provided either to all employees or to none. As explained in Coughlan, Gates, and Myung (2014), however, this approach is horribly inefficient in that most non-monetary incentives (in fact, all NMIs we investigated) are valued above cost by only a minority of the population to which they may be provided. The only efficient solution is to provide each NMI, or combination of NMIs, to only those individuals who actually value it above cost. This is achieved in all variations of CRAM described in Appendix C and, therefore, we could arguably compare the described variations, for example, however that would seem to be outside the scope of the current paper. Finally, in the spirit of market design, we will compare CRAM to the mechanism that is currently used: Cash-based compensation.

VIII. Employer Cost

First we compare CRAM's performance to our benchmark alternative, a monetary retention auction, in terms of overall employer cost.

LEMMA 6: For any $i \in I$ and any set of NMIs *N*, the employer's cost to satisfy employee *i*'s optimal bid under CRAM is less than or equal to the cost to satisfy employee *i*'s optimal bid under a uniform-price monetary retention auction. In other words, $b_i \leq \hat{m}_i$.

The above lemma indicates that all employees will submit weakly lower-cost bids under CRAM than under a uniform-price monetary retention auction. Moreover, from the logic of the proof, we can say the following: As long as an employee values some set of NMIs greater than the cost to provide that set of NMIs, the employee will submit a strictly lower-cost bid under CRAM than under a uniform-price monetary retention auction. An employee will never (optimally) submit a higher-cost bid under CRAM than under the monetary auction, and the only scenario in which an employee would submit bids of identical cost under each mechanism is when no combination of NMIs provides value greater than the cost to provide that combination of NMIs.

Knowing that employees will optimally submit weakly lower-cost bids under CRAM than under the monetary retention auction, it is not surprising that the actual total retention cost under CRAM is less than the cost under a monetary auction. The following Theorem formalizes this result.

THEOREM 2: Given any set of employees *I*, any number of retainees $q \leq |I|$, and any set of NMIs *N*, the cost-per-retainee under CRAM is less than or equal to the cost-per-retainee under a monetary retention auction. In other words, $b^* \le \hat{m}^*$.

Theorem 2 indicates that CRAM will weakly outperform a uniform-price monetary retention auction in terms of minimizing employer cost. Also, recall that such a uniform-price retention auction itself weakly outperforms other cash-only retention mechanism, such as the "posted-price" approach currently employed by the U.S. military.

Furthermore, CRAM also offers employers the practical advantage of being able to mitigate risk in the presence of uncertainty by raising the cost or "price tags" for NMIs. As noted previously, the employer could set high costs for each NMI, perhaps well above actual costs, in which case only employees with exceedingly high valuations would chose NMIs. For all other employees, CRAM would be equivalent to the benchmark monetary auction. At the extreme, if NMI costs were set higher than all employee valuations, CRAM simply converts to a monetary retention auction for all parties.

While Theorem 2 states that CRAM will always cost no more than the monetary auction, it is important to note that there are many scenarios in which CRAM will indeed cost strictly less than the monetary auction. Example 1 in **Appendix B** illustrates one such scenario. Nonetheless, while an employer will always (weakly) prefer CRAM, Example 1 also illustrates that the employees are not necessarily better off under CRAM. This question of employee utility is the topic of the next section.

IX. Employee Utility

In this section, we derive the conditions under which CRAM will increase or decrease employees' utility relative to the benchmark uniform-price monetary auction. First of all, Example 1 of **Appendix B** showed that CRAM could produce both lower combined employee surplus (40 vs. 60) and lower total utility

(160 vs. 180) than a monetary retention auction. Hence, CRAM could improve the outcome for the employer while worsening the outcome for the average employee. Example 2, also in **Appendix B**, demonstrates that, on the other hand, *all* parties (employer and all employees) could actually gain under CRAM. In other words, under some conditions, CRAM will not only lower employer cost, but will also strictly increase total employee utility and surplus relative to the benchmark monetary retention auction.

With these examples demonstrating a range of possibilities, we now derive general conditions under which an employee will be better-off, worse-off, or the same under CRAM as opposed to a monetary retention auction. In order to do so, we introduce some additional notation. For a given set of NMIs *N*, a given set of employees *I*, and a number of retainees $q \leq |I|$, let:

C denotes set of employees retained under CRAM;

$\bar{C}$ denotes set of employees not retained under CRAM;

M denotes set of employees retained under a monetary auction; and

$\bar{M}$ denotes set of employees not retained under a monetary auction.

Based on the retention outcome for an employee under the two mechanisms, each individual will thus fall into one of four categories: (1) $\bar{C} \cap \bar{M}$, (2) $\bar{C} \cap M$, (3) $C \cap \bar{M}$, or (4) $C \cap M$. In what follows, we investigate the preferences of employees in each of these four categories and also provide a condition under which CRAM represents a Pareto-improvement for all parties.

First, if $i \in \bar{C} \cap \bar{M}$, employee *i* is retained under neither mechanism and will receive his reservation utility, r_i, regardless. Hence, such an employee is indifferent between the two mechanisms.

Second, if $i \in \bar{C} \cap M$, employee *i* would be retained under a monetary auction but not under CRAM, and thus will (weakly) prefer the monetary auction.

To see this, note that employees not retained under CRAM receive their reservation utility, r_i, while employees retained under the monetary auction receive the cutoff monetary bid amount of $\hat{m}^*$. As noted previously, however, with optimal bidding, $\hat{m}^* \geq \hat{m}_i = r_i$. Hence, employees in this category will (weakly) prefer the monetary auction.

Third, if $i \in C \cap \bar{M}$, employee i would be retained under CRAM but not under a monetary auction, and hence will (weakly) prefer CRAM. Such employees receive only their reservation value, r_i, under a monetary auction, but receive utility $U_i\left(P_i\right)$ under CRAM. As established previously, however, we have $U_i\left(P_i\right) \geq r_i$ for any employee i retained under CRAM. Hence, employees in this category (weakly) prefer CRAM to the monetary auction

Finally, if $i \in C \cap M$, employee i is retained under both mechanisms and his preference between mechanisms will depend upon a number of considerations. As noted previously, CRAM's first excluded bid cost is weakly smaller than the monetary auction's first excluded bid, or $b^* \leq \hat{m}^*$. Hence, the average cost of employee compensation is lower under CRAM than under the monetary retention auction. But is this decrease in cost to the employer offset by an increase in value to the retained employee? The following proposition tells us that any employee retained under both mechanisms will weakly prefer CRAM to the retention auction if his maximum NMI surplus is at least as large as the decrease in the cost of the first excluded bid.

PROPOSITION 1: Any employee $i \in M \bigcap C$ weakly prefers CRAM to the monetary retention auction if and only if $\max_{S \subseteq N} surplus(i,S) \geq \hat{m}^* - b^*$, with the preference being strict if and only if $\max_{S \subseteq N} surplus(i,S) > \hat{m}^* - b^*$.

In interpreting Proposition 1, first note that, under the monetary retention auction, the *cost* of each retained employee's incentive package is $\hat{m}^*$, which is also precisely equal to the *value* of this package. In contrast, under CRAM the *value* of a retained employee i's incentive package exceeds the cost of that package by exactly $\max_{S \subseteq N} surplus(i,S)$. At the same time, the *cost* of each retained employee's incentive package decreases by $\hat{m}^* - b^*$ when changing from a monetary retention auction to CRAM (and note that this difference itself essentially reflects the maximum NMI surplus achieved at the margin). Therefore, Proposition 1 says that any employee i retained under both mechanisms will prefer a switch to CRAM from a monetary retention auction so long as what he or she gains from the switch, namely $\max_{S \subseteq N} surplus(i,S)$, is greater than what he or she loses from the switch, which is $\hat{m}^* - b^*$.

An important implication of Proposition 1 is that, if the inequality holds for the entire group of employees retained under either mechanism, switching from a monetary retention auction to CRAM would be a Pareto-improvement for the entire group, assuming at least one member of the group has an NMI surplus strictly greater than the decrease in the cost of the first excluded bid. Formally, if $\min_{i \in M \cap C} \max_{S \subseteq N} surplus(i,S) \geq \hat{m}^* - b^*$ and $\exists i \in M \cap C$, $\exists S \subseteq N$ such that $surplus(i,S) > \hat{m}^* - b^*$, then some members of this group strictly prefer CRAM while no members strictly prefer the monetary auction.

Hence, we have shown that employee preferences between the two mechanisms depend upon both the retention outcome for the employee under each mechanism as well as the magnitude of the NMI surplus relative to the difference in cutoff costs. Table 1 summarizes these results. Moreover, while the relative frequency of scenarios in which switching to CRAM would be a Pareto improvement for all players is an empirical question, the information in Table 1 also allows us to at least identify some sufficient conditions.

Table 1. Retention Mechanism Preference by Employee Category

Employee Category	Mechanism Preference
$i \in \bar{C} \cap \bar{M}$	No Preference
$i \in \bar{C} \cap M$	Monetary Auction
$i \in C \cap \bar{M}$	CRAM
$i \in C \cap M$	CRAM $\Leftrightarrow \max_{S \subseteq N} surplus(i,S) \geq \hat{m}^* - b^*$ Monetary Auction $\Leftrightarrow \max_{S \subseteq N} surplus(i,S) < \hat{m}^* - b^*$

For example, since any employee $i \in \bar{C} \cap M$ will at least weakly prefer the monetary auction, scenarios in which this particular set of employees is empty (i.e., $\bar{C} \cap M = \varnothing$) would weigh in favor of CRAM. This occurs if and only if the same set of employees are retained under each mechanism (i.e., $C = M$).

Furthermore, for CRAM to be (weakly) preferred by all retained employees, Table 1 also tells us that the maximum NMI surplus for each such employee must be at least as large as the difference between the cutoff costs under the two mechanisms. We now provide two corollaries, which provides a Pareto-improvement by switching from the monetary retention auction to CRAM.

COROLLARY 3: If $b^* = \hat{m}^*$ and at least one employee $i \in C$ has $\max_{S \subseteq N} surplus(i,S) > 0$, then there is a Pareto-improvement between all parties (retained employees, unretained employees, and the employer) by switching from the Monetary Retention Auction to CRAM.

COROLLARY 4: If $\max_{S \subseteq N} surplus(i,S) > \max_{S \subseteq N} surplus(j,S)$ for all $i \in M$ and $j \notin M$, then $M = C$ and there is a Pareto-improvement between all parties by switching from the Monetary Retention Auction to CRAM.

Corollary 3 and 4 provides two conditions that provide a Pareto-improvement in our environment. In other words, among the entire set of employees (retained and unretained) and the employer, CRAM is weakly preferred by all parties since it weakly increases employee's utility and weakly decreases the employer's cost. One condition is when the cutoff cost between the two mechanisms are the same. Second is the case where employees retained under the monetary retention auction generate a strictly higher NMI surplus than the unretained. The second condition also states that the same group of people will be retained by switching from the monetary retention auction to CRAM. We may relax the assumption of the strict inequality of NMI surplus between the retained and unretained with only one person having strict inequality to also have a Pareto-improvement. However, then we cannot guarantee that $M = C$, due to tie breaking rules and those who were tied at q and $q+1$st bid on the monetary auction. The question of social welfare in general will be addressed in the next section.

X. Social Welfare

In the previous two sections, we demonstrated that CRAM generates lower employer costs than a monetary retention auction, but that it may also lower employee utility. The critical remaining question, therefore, is whether CRAM improves social welfare. In particular, are CRAM's cost savings greater than or equal to any potential reduction in employee utility? In this section, we prove that the answer to this question is "Yes."

First, we define social welfare in this environment as total employee utility (both retained and unretained) minus total employer costs. This definition recognizes that we have explicitly defined utility functions for the employees, but we have not done so for the employer. We have only said that the employer's objective is to retain q employees at the lowest possible cost. Therefore, it is natural to measure social welfare as utility minus cost in this context.

Let W_M denote the total social welfare under a monetary retention auction while W_C denotes the total social welfare under CRAM. Then, summing up total employee utility (both retained and unretained) and subtracting employer cost under each mechanism, we have: $W_M = \sum_{i \in M} \left(U_i(\hat{m}^*) - \hat{m}^* \right) + \sum_{i \in \overline{M}} \left(r_i \right)$ and $W_C = \sum_{i \in C} \left(U_i(P_i) - b^* \right) + \sum_{i \in \overline{C}} \left(r_i \right)$.

THEOREM 3: For any I, $q \leq |I|$, and N, total social welfare is weakly higher under CRAM than under a monetary retention auction. In particular, we have $W_C = \sum_{i \in C} \left(U_i(P_i) - b^* \right) + \sum_{i \in \overline{C}} \left(r_i \right) \geq \sum_{i \in M} \left(U_i(\hat{m}^*) - \hat{m}^* \right) + \sum_{i \in \overline{M}} \left(r_i \right) = W_M$.

Theorem 3 thus indicates that, not only does CRAM reduce employer cost, it also increases total social welfare. Hence, while there are some conditions in which CRAM might lower employee utility (relative to the monetary retention auction) as stated in the previous section, in net, the gain in social welfare outweighs any loss in the welfare. Moreover, there are many conditions in which CRAM will both reduce employer cost and increase employee utility.

XI. Summary and Issues for Further Research

Employers often have an opportunity to offer employees non-monetary compensation that employees value well in excess of the employer's cost of provision. However, employee preferences across NMIs are diverse. What is valuable to some has little or no value to others. As stated earlier, surveys of military service members illustrate the difficulty of identifying any NMI that has significant value for even 50 percent of the service members surveyed, but also show that approximately 80 percent of the surveyed service members expressed a significant value for at least one NMI. These surveys show that employers could reduce compensation costs by relying more heavily on NMIs. However, the key to exploiting this potential is personalizing the employees' NMI packages to reflect their individual preferences.

CRAM provides a mechanism to accomplish this objective when setting employee retention bonuses, though it can easily be extended to voluntary separation incentives and other areas of employee compensation. CRAM is a reverse uniform price auction that combines monetary compensation with the costs of an individualized set of NMIs to create a single total retention cost parameter. CRAM retains the least expensive total cost employees, providing each a compensation package with a cost equal to the cost of the first excluded bid. Each employee receives their requested NMIs and a cash bonus equal to the total cost of the first excluded bid minus the total cost of that employee's package of NMIs.

This paper has demonstrated that CRAM is a dominant strategy incentive compatible mechanism. The weakly optimal strategy for any employee is to select the set of NMIs that maximize surplus value (the employee's value minus the total provision costs) and include a cash request so that the bid's total value to the employee equals the employee's reservation value of employment. Compared to a reverse uniform price monetary auction, CRAM is never more expensive than the purely monetary compensation, and often less expensive. Furthermore, CRAM provides at least as much, and often greater, total social welfare compared to a monetary auction.

However, the employee outcomes under CRAM are more complicated. This is most obvious considering that potentially different sets of employees are retained under CRAM and a monetary auction. In fact, some employees will be better off under CRAM, including those retained under CRAM but not retained in a monetary auction; some employees are better off under a monetary auction, including those retained under a monetary auction but not under CRAM; some employees are indifferent, including those not retained under either auction; and some may be worse off or better off depending on how much the cost of first excluded bid has changed, including those who were obtained in both CRAM and monetary auction.

Considering the expected reduction in employer cost and increase in total social welfare, in conjunction with the truth-revealing attributes CRAM offers, CRAM appears to be an attractive approach to setting retention compensation in the military personnel system, and provides potential for a much broader range of applications. This is particularly important when there is an increase in pressure on the military budget.

One concern regarding both the current posted-price military retention process and CRAM or a simple monetary auction, observes that all three process retain the least expensive employees (most willing to serve or work). There may be cases where an employer would pay a premium to retain higher quality employees or to increase the flexibility of the type of employees retained. Although there is no a priori reason to think that there is a relationship between quality of service member and their reservation value, Quality Adjusted Uniform Price Auction (QUAD) (Myung 2013), is a mechanism developed precisely to control for quality of employees retained.[18] QUAD improves the employer's ability to control cost and the number of employees retained, and also the quality of employees retained while still being a dominant strategy incentive compatible mechanism. Myung (2013) argued that, for the DoD's retention and separation

[18] Myung (2013) does not find correlation between quality and reservation price.

problem, there are three important positive characteristics that the end user to should be able to control and adjust. These three are 1) cost of retention (cost), 2) number of employees being retained (quantity), and 3) quality of employees being retained (quality). CRAM can be modified to incorporate a QUAD-like mechanism process as well.

The ultimate goal for our research stream is to integrate market-based processes throughout the military personnel system, and apply these mechanisms more broadly as appropriate.

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Appendix A. Proofs

LEMMA 1: For any set of available NMIs N and any employee i, $\max_{S\subseteq N}\left(surplus\left(i,S\right)\right)\geq 0$ and $\min_{S\subseteq N}\left(surplus\left(i,S\right)\right)\leq 0$.

PROOF: Note that the set of all potential NMI packages, $S\subseteq N$, includes the empty set, $\varnothing$, for which $v_i\left(\varnothing\right)=0$ and $cost\left(\varnothing\right)=0$. Therefore, $surplus\left(i,\varnothing\right)$ = $v_i\left(\varnothing\right)-cost\left(\varnothing\right)$ = 0. Thus, it must be the case that $\max_{S\subseteq N}\left(surplus\left(i,S\right)\right)\geq 0$ and that $\min_{S\subseteq N}\left(surplus\left(i,S\right)\right)\leq 0$. ■

LEMMA 2: For any employee *i* retained under CRAM, $m_i^*\geq m_i$.

PROOF: Recall that $m_i^* = b^*-cost\left(S_i\right)$ and that $b_i=m_i+cost\left(S_i\right)$ or, in other words, $m_i=b_i-cost\left(S_i\right)$. Because employee *i* was retained, we must have $b^*\geq b_i$, which implies $b^*-cost\left(S_i\right)\geq b_i-cost\left(S_i\right)$ and, therefore, $m_i^*\geq m_i$. ■

LEMMA 3: For any employee *i* retained under CRAM, $U_i\left(P_i\right)\geq U_i\left(B_i\right)$.

PROOF: Recall that $P_i=\left(m_i^*,S_i\right)$ and that $B_i=\left(m_i,S_i\right)$. Hence, $U_i\left(P_i\right)=U_i\left(m_i^*,S_i\right)=v_i\left(S_i\right)+m_i^*$ and $U_i\left(B_i\right)=U_i\left(m_i,S_i\right)=v_i\left(S_i\right)+m_i$. In other words, $U_i\left(P_i\right)=U_i\left(B_i\right)+m_i^*-m_i$. Because, as explained in Lemma 2, we have that $m_i^*\geq m_i$, it must also be the case that $U_i\left(P_i\right)\geq U_i\left(B_i\right)$. ■

LEMMA 4: Given any reservation value $r_i\in\mathbb{R}$ and any set of NMIs $S_i\subseteq N$, bid $B_i=\left(m_i,S_i\right)$ maximizes employee *i*'s utility under CRAM for any set of competing bids, B_{-i}, if and only if $m_i=r_i'=r_i-v_i\left(S_i\right)$.

PROOF: The proof of this lemma follows the structure of the standard proof for the incentive-compatibility of a second-price auction. In each possible scenario, we demonstrate that submitting any bid $B_i' = (m', S_i)$, where $m' \neq r_i'$, will never generate greater utility than the bid $B_i = (r_i', S_i)$ and will sometimes yield strictly lower utility.

Before proceeding, note that the cost of bid $B_i = (r_i', S_i)$ is given by b_i' = $cost(B_i') = m' + cost(S_i)$, while the cost of bid $B_i = (r_i', S_i)$ is given by b_i = $cost(B_i)$ = $r_i' + cost(S_i)$ = $r_i - v_i(S_i) + cost(S_i)$ = $r_i - surplus(i, S_i)$. We will continue to denote the "cutoff cost" by b^*.

<u>Scenario 1:</u> Employee *i* retained with bid $B_i = (r_i', S_i)$

<u>Sub-scenario 1A:</u> Employee *i* also retained with bid $B_i' = (m', S_i)$

Because employee *i* is still retained with a bid of $B_i' = (m', S_i)$ in this sub-scenario, it must be the case, whether or not $m' > r_i'$ or $m' < r_i'$, that b_i' = $m' + cost(S_i) \leq b^*$. Employee *i* will also still receive the same retention package $P_i = (b^* - cost(S_i), S_i)$, since both the set of NMIs requested, S_i, and the cutoff cost, b^*, remain unchanged. Thus, in this sub-scenario, employee *i* will *not* do better by submitting a bid of $B_i' = (m', S_i)$.

<u>Sub-scenario 1B:</u> Employee *i* not retained with bid $B_i' = (m', S_i)$

Employee i not retained with a bid of $B_i' = (m', S_i) \Rightarrow b_i' \geq b^*$ and this bid yields only the reservation value of r_i. On the other hand, employee i retained with a bid of $B_i = (r_i', S_i) \Rightarrow b_i \leq b^*$ and this bid yields a retention package $P_i = (b^* - cost(S_i), S_i)$ generating utility $U_i(P_i) = U_i(b^* - cost(S_i), S_i) = v_i(S_i) + b^* - cost(S_i) = b^* + surplus(i, S_i) \geq b_i + surplus(i, S_i) = r_i$. Because $U_i(P_i) \geq r_i$, employee i will *not* do better by submitting a bid of $B_i' = (m', S_i)$ in this sub-scenario.

Moreover, $b_i' \geq b^* \geq b_i \Rightarrow m' + cost(S_i) \geq b^* \geq r_i' + cost(S_i) \Rightarrow m' \geq r_i'$. Because $m' \neq r_i'$, however, it must be the case that $m' > r_i' \Rightarrow m' + cost(S_i) > r_i' + cost(S_i) \Rightarrow b_i' > b_i$. Hence, there exists some set of competing bids *B*-*i* such that $b_i' > b^* > b_i$. In that case, $U_i(P_i) = b^* + surplus(i, S_i) > b_i + surplus(i, S_i) = r_i$, so that the utility generated by bid $B_i = (r_i', S_i)$ is strictly greater than the utility generated by bid $B_i' = (m', S_i)$.

<u>Scenario 2:</u> Employee i not retained with bid $B_i = (r_i', S_i)$

<u>Sub-scenario 2A:</u> Employee i also not retained with bid $B_i' = (m', S_i)$

In Scenario 2, employee i not retained with bid $B_i = (r_i', S_i) \Rightarrow b_i \geq b^*$ and such a bid yields only the reservation value of r_i. Because employee i is also not retained with a bid of $B_i' = (m', S_i)$ in Sub-scenario 2A, it must be the case, whether or not $m' > r_i'$ or $m' < r_i'$, that $b_i' = m' + cost(S_i) \geq b^*$. Because employee i still receives only his reservation value of r_i with either bid in this sub-scenario, employee i can once again *not* do better by submitting a bid of $B_i' = (m', S_i)$.

<u>Sub-scenario 2B:</u> Employee *i* retained with bid $B_i' = (m', S_i)$

Employee *i* retained with bid of $B_i' = (m', S_i) \Rightarrow b_i' = m' + cost(S_i) \leq b^*$ and such a bid yields retention package $P_i = (b^* - cost(S_i), S_i)$, giving utility $U_i(P_i) = b^* + surplus(i, S_i)$. Employee i not retained with bid $B_i = (r_i', S_i) \Rightarrow$ $b_i \geq b^* \Rightarrow U_i(P_i) = b^* + surplus(i, S_i) \leq b_i + surplus(i, S_i) = r_i$. Because $U_i(P_i) \leq r_i$, employee *i* will *not* do better by submitting a bid of $B_i' = (m', S_i)$ in this sub-scenario.

Moreover, $b_i' \leq b^* \leq b_i \Rightarrow m' + cost(S_i) \leq b^* \leq r_i' + cost(S_i) \Rightarrow m' \leq r_i'$. Because $m' \neq r_i'$, however, it must be the case that $m' < r_i' \Rightarrow m' + cost(S_i) < r_i' + cost(S_i) \Rightarrow b_i' < b_i$. Hence, there exists some set of competing bids B_{-i} such that $b_i' < b^* < b_i$. In that case, $U_i(P_i) = b^* + surplus(i, S_i) < b_i + surplus(i, S_i) = r_i$, so that the utility generated by bid $B_i = (r_i', S_i)$ is strictly greater than the utility generated by bid $B_i' = (m', S_i)$.

Summarizing over all scenarios, we have demonstrated that submitting any bid $B_i' = (m', S_i)$, where $m' \neq r_i'$, will never generate greater utility than the bid $B_i = (r_i', S_i)$ and will sometimes yield strictly lower utility. ■

LEMMA 5: For any reservation value $r_i \in \mathbb{R}$ and any monetary bid $m_i \in \mathbb{R}$, employee *i*'s utility from being retained under CRAM, $U_i(P_i)$, will be maximized if and only if he submits a bid $B_i = (m_i, S_i)$ where $S_i \in \operatorname{argmax}_{S \subseteq N}(surplus(i,S))$.

PROOF: If retained, employee *i*'s compensation package will be $P_i = (m_i^*, S_i) = (b^* - cost(S_i), S_i)$, which provides utility of $U_i(P_i) = v_i(S_i) + m_i^* = v_i(S_i) - cost(S_i) + b^* = surplus(i, S_i) + b^*$. Because the cutoff cost b^* is independent of employee *i*'s non-monetary bid S_i (and, incidentally, independent of his monetary bid m_i as well), employee *i* maximizes $U_i(P_i)$ if and only if he chooses S_i to maximize $surplus(i, S_i)$ or, in other words, if he chooses S_i such that $S_i \in \operatorname{argmax}_{S \subseteq N}(surplus(i,S))$. ■

THEOREM 1: (Dominant Strategy Incentive Compatibility Theorem) Given any reservation value $r_i \in \mathbb{R}$, bid $B_i = (m_i, S_i)$ maximizes employee *i*'s utility under CRAM for any possible sets of competing bids, B_{-i}, if and only if $m_i = r_i' = r_i - v_i(S_i)$ and $S_i \in \operatorname{argmax}_{S \subseteq N}(surplus(i,S))$.

PROOF: From Lemma 4, we know that, given any set of NMIs $S_i \subseteq N$, bid $B_i = (m_i, S_i)$ maximizes employee *i*'s utility for any set of competing bids,

B_{-i}, if and only if $m_i = r_i' = r_i - v_i(S_i)$. Hence, we must only prove that, given such a monetary bid m_i, an NMI bid S_i maximizes employee *i*'s utility under all scenarios if and only if $S_i \in \text{argmax}_{S \subseteq N}(surplus(i,S))$.

In order to do so, we will show that any alternative bid $B_i' = (m_i', S_i')$, with $S_i' \notin \text{argmax}_{S \subseteq N}(surplus(i,S))$ will never generate greater utility than the bid $B_i = (r_i', S_i)$ with $S_i \in \text{argmax}_{S \subseteq N}(surplus(i,S))$ and will sometimes yield strictly lower utility. First of all, we know from Lemma 4 that utility from this alternative bid $B_i' = (m_i', S_i')$ is maximized if and only if $m_i' = r_i - v_i(S')$, so we can assume this to be true of m_i'.

Note that the cost of bid $B_i' = (m_i', S_i')$ is given by b_i' = $m_i' + cost(S_i')$ = $r_i - v_i(S') + cost(S_i')$ = $r_i - surplus(i, S_i')$. $S_i' \notin \text{argmax}_{S \subseteq N}(surplus(i,S))$ while $S_i \in \text{argmax}_{S \subseteq N}(surplus(i,S))$ $\Rightarrow$ $surplus(i, S_i')$ < $surplus(i, S_i)$ $\Rightarrow$ b_i' = $m_i' + cost(S_i')$ = $r_i - v_i(S') + cost(S_i')$ = $r_i - surplus(i, S_i')$ > $r_i - surplus(i, S_i)$ = b_i.

<u>Scenario 1:</u> Employee *i* not retained with bid $B_i = (m_i, S_i)$.

Employee *i* not retained with bid $B_i = (m_i, S_i)$ $\Rightarrow$ $b_i \geq b^*$. Thus, $b_i' > b_i$ $\Rightarrow$ $b_i' > b^*$ $\Rightarrow$ employee *i* also not retained with bid $B_i' = (m_i', S_i')$ $\Rightarrow$ employee *i* receives reservation value r_i with either bid $\Rightarrow$ bid $B_i' = (m_i', S_i')$ does not generate greater utility than bid $B_i = (r_i', S_i)$.

<u>Scenario 2:</u> Employee *i* retained with bid $B_i = (m_i, S_i)$

Employee i retained with bid $B_i = (m_i, S_i) \Rightarrow b^* \geq b_i$. Thus, $b_i' > b_i \Rightarrow$ employee *i* may or may not retained with bid $B_i' = (m_i', S_i')$.

<u>Sub-scenario 2A:</u> Employee *i* also retained with bid $B_i' = (m_i', S_i')$

Lemma 5 states that employee *i*'s utility from being retained under CRAM will be maximized if and only if he submits a bid $B_i = (m_i, S_i)$ where $S_i \in \text{argmax}_{S \subseteq N}(surplus(i,S))$. Thus, the utility generated by bid $B_i = (r_i', S_i)$ would always be strictly greater than the utility generated by the alternative bid $B_i' = (m_i', S_i')$ under this sub-scenario.

<u>Sub-scenario 2B:</u> Employee *i* not retained with bid $B_i' = (m_i', S_i')$

Employee *i* retained with bid $B_i = (m_i, S_i) \Rightarrow b^* \geq b_i$ and employee *i* receives compensation package $P_i = (m_i^*, S_i) = (b^* - cost(S_i), S_i)$, which provides utility of $U_i(P_i) = v_i(S_i) + m_i^* = v_i(S_i) - cost(S_i) + b^* = surplus(i, S_i) + b^* \geq b_i + surplus(i, S_i) = r_i$. Employee *i* not retained with bid $B_i' = (m_i', S_i') \Rightarrow$ employee *i* receives reservation wage r_i. $U_i(P_i) \geq r_i \Rightarrow$ the alternative bid $B_i' = (m_i', S_i')$ with $S_i' \notin \text{argmax}_{S \subseteq N}(surplus(i,S))$ can not generate greater utility than bid $B_i = (r_i', S_i)$ under this sub-scenario.

Moreover, $b_i' > b_i \Rightarrow$ there exists some set of competing bids B_{-i} such that $b_i' > b^* > b_i$ in this sub-scenario. For any such b^*, $U_i(P_i) = surplus(i, S_i) + b^* > b_i + surplus(i, S_i) = r_i$, so that the utility generated by bid

$B_i = \left(r_i', S_i\right)$ would be strictly greater than the utility generated by bid $B_i' = \left(m_i', S_i'\right)$.

Summarizing over all scenarios, we have demonstrated that submitting any bid $B_i' = \left(m_i', S_i'\right)$ with $S_i' \notin \text{argmax}_{S \subseteq N}\left(surplus\left(i, S\right)\right)$ will never generate greater utility than the bid $B_i = \left(r_i', S_i\right)$ and will sometimes yield strictly lower utility. ■

COROLLARY 1: In the dominant strategy equilibrium in **Theorem 1,** the cost-to-retain associated with the optimal bid of any employee *i* under CRAM is given by $b_i = r_i - \max_{S \subseteq N}\left(surplus\left(i, S\right)\right)$.

PROOF: Recall that the cost-to-retain associated with a bid $B_i = \left(m_i, S_i\right)$ from employee *i* is given by $b_i = m_i + cost\left(S_i\right)$. If this bid includes the optimal monetary bid of $m_i = r_i - v_i\left(S_i\right)$, this cost becomes $b_i = r_i - v_i\left(S_i\right) + cost\left(S_i\right) = r_i - surplus\left(i, S_i\right)$. Finally, if employee *i* also selects the optimal set of NMIs, such that $S_i \in \text{argmax}_{S \subseteq N}\left(surplus\left(i, S\right)\right)$, then the cost-to-retain associated with bid $B_i = \left(m_i, S_i\right)$ becomes $b_i = r_i - \max_{S \subseteq N}\left(surplus\left(i, S\right)\right)$. ■

COROLLARY 2: In the dominant strategy equilibrium in **Theorem 1**, any employee *i* who submits the optimal bid under CRAM will receive a retention package P_i, generating utility $U_i\left(P_i\right) = b^* + \max_{S \subseteq N}\left(surplus\left(i, S\right)\right)$ if retained.

PROOF: If retained with a bid of $B_i = \left(m_i, S_i\right)$, employee *i* will receive retention package $P_i = \left(b^* - cost\left(S_i\right), S_i\right)$, which provides utility of

$U_i(P_i) = b^* - cost(S_i) + v_i(S_i) = b^* + surplus(i, S_i)$. If employee i has selected the optimal set of NMIs, such that $S_i \in \text{argmax}_{S \subseteq N}(surplus(i, S))$, then this utility becomes $U_i(P_i) = b^* + \max_{S \subseteq N}(surplus(i, S))$. ■

LEMMA 6: For any $i \in I$ and any set of NMIs N, the employer's cost to satisfy employee i's optimal bid under CRAM is less than or equal to the cost to satisfy employee i's optimal bid under a uniform-price monetary retention auction. In other words, $b_i \leq \hat{m}_i$.

PROOF: As described in the text, under a uniform-price monetary retention auction, it is a dominant strategy for each bidder to truthfully reveal his or her reservation value r_i by bidding $\hat{m}_i = r_i$. Thus, the minimum cost to retain employee i under this monetary retention auction is equal to r_i.

Under CRAM, on the other hand, we know from Corollary 1 that the cost-to-retain associated with the optimal bid of any employee i is given by b_i = $r_i - \max_{S \subseteq N}(surplus(i, S))$. From Lemma 1, we know $\max_{S \subseteq N}(surplus(i, S)) \geq 0$ $\Rightarrow$ $b_i = r_i - \max_{S \subseteq N}(surplus(i, S)) \leq r_i$. Hence, for any employee i, the employer's cost to satisfy his optimal bid under CRAM is less than or equal to the cost to satisfy his optimal bid under a monetary retention auction. ■

THEOREM 2: Given any set of employees I, any number of retainees $q \leq |I|$, and any set of NMIs N, the cost-per-retainee under CRAM is less than or equal to the cost-per-retainee under a monetary retention auction. In other words, $b^* \leq \hat{m}^*$.

PROOF: Lemma 6 tells us that, for all $i \in I$, $b_i \leq \hat{m}_i = r_i$. Therefore, the lowest-cost bid under CRAM must cost less than (or the same as) the lowest-cost bid under the monetary auction, the highest-cost bid under CRAM must cost less than (or the same as) the highest-cost bid under the monetary auction, and the n^{th} lowest-cost bid under CRAM must cost less than (or the same as) the n^{th} lowest-cost bid under the monetary auction for any $n \in \{0,1,...,|I|\}$. Therefore, the cutoff cost b^*, which is equal to the cost of the $(q+1)^{st}$ lowest-cost bid under CRAM, must be less than (or the same as) $\hat{m}^*$, which is the cost of the $(q+1)^{st}$ lowest-cost bid under the monetary auction. In other words, $b^* \leq \hat{m}^*$. ■

PROPOSITION 1: Any employee $i \in M \cap C$ weakly prefers CRAM to the monetary retention auction if and only if $\max_{S \subseteq N} surplus(i,S) \geq \hat{m}^* - b^*$, with the preference being strict if and only if $\max_{S \subseteq N} surplus(i,S) > \hat{m}^* - b^*$.

PROOF: Recall that $b_i \leq \hat{m}_i$ (Lemma 6) and $b^* \leq \hat{m}^*$ (Theorem 2). For any employee $i \in M \cap C$, utility under CRAM in equilibrium is $U_i(P_i) = m_i^* + v_i(S_i) = b^* + \max_{S \subseteq N} surplus(i,S)$ and utility under the monetary auction in equilibrium is $U_i(\hat{m}^*) = \hat{m}^*$. So, $U_i(P_i) \geq U_i(\hat{m}^*) \Leftrightarrow b^* + \max_{S \subseteq N} surplus(i,S) \geq \hat{m}^*$ $\Leftrightarrow \max_{S \subseteq N} surplus(i,S) \geq \hat{m}^* - b^*$ and the preference is strict whenever $U_i(P_i) > U_i(\hat{m}^*) \Leftrightarrow \max_{S \subseteq N} surplus(i,S) > \hat{m}^* - b^*$. ■

COROLLARY 3: If $b^* = \hat{m}^*$ and at least one employee $i \in C$ has $\max_{S \subseteq N} surplus(i,S) > 0$, then there is a Pareto-improvement between all parties (retained employees, unretained employees, and the employer) by switching from the Monetary Retention Auction to CRAM.

PROOF: First, divide the employees into two sets: $M^* = \{i \in I \mid \hat{m}_i < \hat{m}^*\}$ and $\bar{M}^* = \{i \in I \mid \hat{m}_i \geq \hat{m}^*\}$. These sets are different than M and $\bar{M}$ due to ties that may occur when $\hat{m}_i = \hat{m}^*$.

Employees in set M^*: because Lemma 7 states that $b_i \leq \hat{m}_i$, if $b^* = \hat{m}^*$, then any i such that $\hat{m}_i < \hat{m}^*$ are all retained under CRAM: $i \in C$. Hence, $M^* \subseteq C$. Furthermore, since $\hat{m}^* - b^* = 0$, and Lemma 1 states $\max_{S \subseteq N} surplus(i,S) \geq 0$, all employees in M^* weakly prefer CRAM to the monetary retention auction by Proposition 1.

Employees in set $\bar{M}^*$: For any i such that $\hat{m}_i \geq \hat{m}^*$, these individuals were only receiving their reservation values r_i in the monetary retention auction regardless of whether $i \in M$ or $i \notin M$. If $i \in M$, then $\hat{m}_i = \hat{m}^*$, therefore, $U_i(\hat{m}^*) = U_i(\hat{m}_i) = r_i$. If $i \notin M$, then he was not retained and received his reservation value r_i. Therefore, these employees are at least indifferent between CRAM and the monetary retention auction. Furthermore, there may exist some individuals (depending on number of ties retained at $\hat{m}^*$) who were not retained under monetary retention auction but are retained under CRAM, and these individuals will weakly prefer CRAM: $U_i(P_i) \geq r_i$.

Employees in set C: since we assume that at least one employee has $\max_{S \subseteq N} surplus(i,S) > 0$, adding $b^* = \hat{m}^*$ to each side, $b^* + \max_{S \subseteq N} surplus(i,S) > \hat{m}^*$ $\Leftrightarrow U_i(P_i) > U_i(\hat{m}_i)$, there exists at least one who strictly prefers CRAM.

Finally, as for the employer, it will cost $b^* = \hat{m}^*$ to retain the q employees in either mechanism. So the employer is equally well off between either mechanisms.

In sum, there is a Pareto-improvement between the employees, and the employer. ■

COROLLARY 4: If $\max_{S\subseteq N} surplus(i,S) > \max_{S\subseteq N} surplus(j,S)$ for all $i \in M$ and $j \notin M$, then $M = C$ and there is a Pareto-improvement between all parties by switching from the Monetary Retention Auction to CRAM.

PROOF: First, the employer is weakly better off by Theorem 2. As for the employees, by Corollary 1, $b_i = r_i - \max_{S\subseteq N} surplus(i,S)$. In equilibrium, $\hat{m}_i = r_i$, thus $b_i = \hat{m}_i - \max_{S\subseteq N} surplus(i,S)$. Since $\max_{S\subseteq N} surplus(i,S) > \max_{S\subseteq N} surplus(j,S)$ and $\hat{m}_i \le \hat{m}_j$ for all $i \in M$ and $j \notin M$, $b_i < b_j$. Therefore, $M = C$. Employees $j \in \overline{M} = \overline{C}$ will continue to receive their reservation value r_j and are indifferent between the mechanisms. Finally, denote employee a as the $q+1$ st highest bidder in the monetary auction: $\hat{m}_a = \hat{m}^*$. In any situation, given that $\max_{S\subseteq N} surplus(i,S) > \max_{S\subseteq N} surplus(j,S)$, the largest decrease in the cutoff bid between the two mechanisms, $\hat{m}^* - b^*$, come from employee a if $a \in \arg\max_{j\in M} \max_{S\subseteq N} surplus(j,S)$. Hence, assume that a is such employee. Then $\hat{m}^* - b^* = \hat{m}_a - b_a = \max_{S\subseteq N} surplus(a,S) < \max_{S\subseteq N} surplus(i,S) \quad \forall i \in C = M$. By Proposition 1, a Pareto-improvement by employees in $M = C$. Therefore, aggregating the results from the employer and all employees, a Pareto-improvement between all parties. ■

THEOREM 3: For any I, $q \le |I|$, and N, total social welfare is weakly higher under CRAM than under a monetary retention auction. In particular, we have $W_C = \sum_{i\in C}\left(U_i(P_i) - b^*\right) + \sum_{i\in \overline{C}}\left(r_i\right) \ge \sum_{i\in M}\left(U_i(\hat{m}^*) - \hat{m}^*\right) + \sum_{i\in \overline{M}}\left(r_i\right) = W_M$.

PROOF: First, we can rewrite the welfare of each mechanism as:

$$W_M = \sum_{i\in M} \hat{m}^* + \sum_{i\in \bar{M}} r_i - q\hat{m}^* = q\hat{m}^* + \sum_{i\in \bar{M}} r_i - q\hat{m}^* = \sum_{i\in \bar{M}} r_i$$

$$W_C = \sum_{i\in C} U_i(P_i) + \sum_{i\in \bar{C}} r_i - qb^* .$$

Under the optimal bidding strategy, W_C becomes:

$$\begin{aligned} W_C &= \sum_{i\in C}\left[b^* + \max_{S\subseteq N}(surplus(i,S))\right] + \sum_{i\in \bar{C}} r_i - qb^* \\ &= \sum_{i\in C} \max_{S\subseteq N}(surplus(i,S)) + \sum_{i\in \bar{C}} r_i \end{aligned} .$$

First, $\forall i \in C$, $b^* \geq b_i = r_i - \max_{S\subseteq N}\left(surplus(i,S)\right)$ and $\forall j \in \bar{C}$, $b^* \leq b_j =$ $r_j - \max_{S\subseteq N}\left(surplus(j,S)\right)$. Hence, $\forall i \in C$ and $\forall j \in \bar{C}$, we have $r_i - \max_{S\subseteq N}\left(surplus(i,S)\right) \leq r_j - \max_{S\subseteq N}\left(surplus(j,S)\right)$. By Lemma 1, we know that $\forall j \in I$, $\max_{S\subseteq N}\left(surplus(j,S)\right) \geq 0$, thus $\forall i \in C$ and $\forall j \in \bar{C}$, we have $r_i - \max_{S\subseteq N}\left(surplus(i,S)\right) \leq r_j$ or $r_j + \max_{S\subseteq N}\left(surplus(i,S)\right) - r_i \geq$ 0.

Note that the inequality $r_j + \max_{S\subseteq N}\left(surplus(i,S)\right) - r_i \geq$ 0 applies to any pair of employees *i* and *j* such that $i \in C \cap \bar{M}$ and $j \in \bar{C} \cap M$. Moreover, because it must be the case that $\left|C \cap \bar{M}\right| = \left|\bar{C} \cap M\right|$, we can match each unique $i \in C \cap \bar{M}$ to a unique $j \in \bar{C} \cap M$. Summing over all such *i* and *j* pairs, we get $\sum_{i\in \bar{C}\cap M} r_j$ + $\sum_{i\in C\cap \bar{M}}\left[\max_{S\subseteq N}(surplus(i,S)) - r_i\right] \geq$ 0.

Again by Lemma 1, we know that $\forall i \in C \cap M$, we have $\max_{S\subseteq N}\left(surplus(j,S)\right) \geq 0$ and, thus, $\sum_{i\in C\cap M} \max_{S\subseteq N}(surplus(i,S)) \geq$ 0. Combining this with our previous inequality, we get:

$$\sum_{i\in C\cap M} \max_{S\subseteq N}(surplus(i,S)) + \sum_{j\in \bar{C}\cap M} r_j$$
$$+ \sum_{i\in C\cap \bar{M}} \left[\max_{S\subseteq N}(surplus(i,S)) - r_i \right] \geq 0$$
$$\Leftrightarrow \sum_{i\in C} \max_{S\subseteq N}(surplus(i,S)) + \sum_{j\in \bar{C}\cap M} r_j \geq \sum_{i\in C\cap \bar{M}} r_i$$
$$\Leftrightarrow \sum_{i\in C} \max_{S\subseteq N}(surplus(i,S)) + \sum_{j\in \bar{C}} r_j - \sum_{j\in \bar{C}\cap \bar{M}} r_j \geq \sum_{i\in \bar{M}} r_i - \sum_{j\in \bar{C}\cap \bar{M}} r_j$$
$$\Leftrightarrow \sum_{i\in C} \max_{S\subseteq N}(surplus(i,S)) + \sum_{i\in \bar{C}} r_i \geq \sum_{i\in \bar{M}} r_i$$
$$\Leftrightarrow W_C \geq W_M \quad \blacksquare$$

Appendix B. Examples

Example 1: *CRAM vs. Monetary Auction (Lower Cost with Lower Utility)*

Suppose there are three employees, such that $|I| = 3$ and $I = \{1,2,3\}$, and that two of these employees are to be retained (i.e., q = 2). Further, suppose that there is only a single NMI offered under CRAM, such that $|N| = 1$ and $N = \{s\}$, and that this NMI can be provided at a constant marginal cost of 10 $\left(\text{i.e., } cost(s) = 10\right)$.

Finally, suppose that each employee's reservation value (r_i) and value for the NMI offered are as indicated in columns two and three of Table 3. Under these conditions, the remaining columns of Table 3 indicate the optimal NMI choice, resulting NMI surplus, optimal CRAM monetary bid, and resulting CRAM bid cost for each employee.

Table 2. *CRAM vs. Monetary Auction Example 1 – Optimal Bid and Cost*

Employee Number I	Reservation Value r_i	NMI Value $v_i(s)$	NMI Chosen S_i	NMI Surplus $Surplus(i,S_i)$	CRAM Money Bid m_i	CRAM Bid Cost b_i
1	20	0	$\varnothing$	0	20	20
2	40	0	$\varnothing$	0	40	40
3	60	20	s	10	40	50

Because q = 2, we have $b^* = b_{q+1} = b_3 = 50$ under CRAM. Similarly, we have $\hat{m}^* = r_{q+1} = r_3 = 60$ under the uniform monetary retention auction. Thus, $b^* < \hat{m}^*$ and the cost-per-retainee under CRAM is strictly less than the cost-per-retainee under a monetary retention auction in this example.

To further understand how the outcome in this example would differ under CRAM relative to a monetary retention auction, however, consider Table 4, which details the retention decision, utility, and surplus for each employee under the monetary auction and under CRAM.

Table 3. *CRAM vs. Monetary Auction Example 1 – Utility Comparison*

Employee Number *I*	Retained in Monetary Auction?	Utility in Monetary Auction	Surplus in Monetary Auction	Retained under CRAM?	Utility under CRAM	Surplus under CRAM
1	Yes	60	40	Yes	50	30
2	Yes	60	20	Yes	50	10
3	No	60	0	No	60	-
TOTAL	-	180	60	-	160	40

Note that the employer is strictly better off under CRAM in this example, but every employee is <u>not</u> better off. In fact, employees 1 and 2 enjoy greater utility and surplus under the monetary retention auction in this example.

Example 2: *CRAM vs. Monetary Auction (Lower Cost with Higher Utility)*

Assume the same basic situation as in Example 1, but with employee NMI values as given in Table 5.

Table 4. *CRAM vs. Monetary Auction Example 2 – Optimal Bid and Cost*

Employee Number I	Reservation Value r_i	NMI Value $v_i(s)$	NMI Chosen S_i	NMI Surplus $Surplus(i,S_i)$	CRAM Money Bid m_i	CRAM Bid Cost b_i
1	20	20	s	10	0	10
2	40	20	s	10	20	30
3	60	20	s	10	40	50

Because q = 2, we once again have $b^* = b_{q+1} = b_3 = 50$ under CRAM and $\hat{m}^* = r_{q+1} = r_3 = 60$ under the uniform monetary retention auction. Table 6 details the retention decision, utility, and surplus for each employee under the monetary auction and under CRAM for this example.

Table 5. *CRAM vs. Monetary Auction Example 2 – Utility Comparison*

Employee Number I	Retained in Monetary Auction?	Utility in Monetary Auction	Surplus in Monetary Auction	Retained under CRAM?	Utility under CRAM	Surplus under CRAM
1	Yes	60	40	Yes	70	50
2	Yes	60	20	Yes	70	30
3	No	60	0	No	60	-
TOTAL	-	180	60	-	200	80

CRAM once again generates lower employer costs in this example, but it also produces a higher total employee surplus (80 vs. 60) and utility (200 vs. 180) than under a monetary retention auction.

Appendix C. Mechanism Variations

We would like to note that the design presented and evaluated in this paper is but one of several different CRAM variations we have investigated. Perhaps the most notable difference among the various CRAM designs explored is in terms of the NMI allocation method. In particular, the CRAM variation presented in this paper allocates NMIs using what we call the "menu-method," whereas an alternative approach explored is what we call the "bid-method."

With the **menu-method** CRAM, one can think of all the NMIs being presented as a menu of options along with associated costs. Employees select their desired NMIs from this "menu." Furthermore, employees know that they will receive the chosen NMIs if selected for retention. Therefore, the cash incentive each employee requests will be the compensation required in addition to the chosen NMIs. In terms of sequencing under the menu-method, the employees first select the NMIs and then submit a monetary bid.

Under the **bid-method** CRAM, employees are asked to submit two types of bids. The first type of bid is equivalent to the Monetary Retention Auction in which employees submit their required cash compensation amounts for retention. An employee's monetary bid is submitted as if no NMIs were offered, because s/he does not know which set of NMIs s/he will be allocated until after the entire process is complete. The second type of bid (or bids) assigns value to the possible NMI combinations to be allocated. These NMI bid amounts reflect the cash bonus the employee is willing to forgo in exchange for the NMIs if retained. Each retained employee will receive the combination of NMIs that maximizes his/her "NMI surplus," which is the difference between the NMI bid amount and the cost of providing those NMIs (recognizing that NMI surplus may be maximized in some cases by the empty set of no NMIs). Notice that, unlike the menu-method approach, there is no need to disclose the cost of NMIs beforehand in the bid-method CRAM. To decide which employees to retain, the employer then takes each employee's monetary bid and subtracts the maximum NMI surplus (NMI bid minus cost) in order to rank and retain the lowest cost employees.

Depending on the environment, either the menu-method or bid-method variation of CRAM may be preferred. For example, in the presence of significant submodular or supermodular NMI valuations, the menu-method offers a clear advantage. First, in this case, if bid-method CRAM only allows employees to submit bids on individual NMIs and not on the various combinations of NMIs, CRAM faces the "exposure problem" of combinatorial auctions discussed previously. On the other hand, if bid-method CRAM were to allow employees to potentially submit separate bids on each of the $2^{|N|}$ different combinations of NMIs, the mechanism would be subject to both the "communication complexity problem" and (to a lesser degree) the "winner determination problem" discussed – not to mention the practical and cognitive difficulties of asking employees to actually formulate and submit precise bid amounts for each possible NMI combination (or at least all combinations containing even partial complements or substitutes).

In contrast, under the menu-method, the employee is not required to submit separate bids on various NMI combinations (or even on all individual NMIs), but must simply incorporate whatever combinatorial values he/she may have into his or her selection of a single preferred package of NMIs (with the weakly dominant strategy being, as illustrated, to select the NMI package which provides the largest NMI surplus). No matter the number of NMIs offered, menu-method CRAM only requires employees to communicate two things to the auctioneer: (1) A preferred combination of NMIs and (2) a monetary bid amount.

Whereas menu-method CRAM has the advantage in environments with more complex NMI valuations (demand-side complexity), bid-method CRAM may have the advantage in environments with more complex NMI costs (supply-side complexity). As noted, menu-method CRAM requires the employer to calculate and communicate in advance a constant unit cost for each NMI. This can be particularly problematic when NMIs exhibit significant economies (or diseconomies) of scale or scope in their provision, or when certain NMIs are in excess demand due to quantity restrictions. Under bid-method CRAM, however, all such complex NMI cost scenarios are more easily addressed. First, the cost of each NMI does not necessarily need to be known in advance. Furthermore, after eliciting the NMI bids, the employer is able to generate a demand curve and then determine the market-clearing price for the NMIs.

Rather than present both the menu-method and bid-method CRAM variations in this paper, however, we have chosen to focus only on the menu-method CRAM design, primarily because it best matches the characteristics of our target application environment, namely military retention. Menu-method CRAM is preferred to bid-method in this environment given that complexity in NMI values or costs is more significant on the demand-side than on the supply-side, as reported in Coughlan, Gates, and Myung (2014). In particular, our own survey research in this area reveals strong complementarity among certain NMI combinations while employees perceived other sets of NMIs to be strong substitutes. Furthermore, many of the single individual NMIs investigated were highly valued by some percentage of the population, yet no NMI had a positive value for a majority of those surveyed (i.e., fewer than 50% of respondents were willing to sacrifice even $1 of cash incentive for any given NMI). This means that the number of service members who actually value an NMI above the cost to provide is even more limited, reducing concerns that the demand curve for any NMI will intersect the portion of the supply curve that is vertical (due to binding quantity constraints) or even significantly upward-sloping (due to increasing marginal costs at higher quantities). As noted above, the menu-method approach has significant advantages in an environment with such characteristics and, therefore, we have focused on the menu-method CRAM design in this paper.

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